

### Problem Set 14.1: Special relativity explanation questions

1. (a) An inertial reference frame is a region of space which has no net force acting on it – it is not accelerating as would be detected by an accelerometer. In inertial frames velocity is constant  
 (b) Not necessarily - they are both inertial frames but not the same frame. Measurements in one inertial frame can be converted to measurements in another by a simple transformation.
2. The laws of physics are the same in all inertial reference frames.  
 The speed of light in a vacuum is absolute and has the same value in all inertial reference frames.
3. “Proper” time is the time between two simultaneous events as seen and measured by an observer who is stationary relative to the object on which measurements are being made
4. Light is subject to Special Relativity, which says that anything with zero rest mass must travel through space-time at the speed of light. Light has zero rest mass and therefore, according to Special Relativity, must always travel through space-time at the speed of light. A photon has no experience of the passage of time so is everywhere at once.
5. The equation  $E = mc^2$  applies only to particles with rest mass. A photon has energy and momentum but no rest mass and must travel at the speed of light. It has an energy proportional to its frequency but no minimum energy and if it stops moving it ceases to exist.
6. Having different velocities means they must be in different frames of reference. This includes having the same speed but moving in different directions or having different speeds in the same direction. In both of these cases there will be an acceleration because a change in velocity exists. According to special relativity if a frame is non-inertial it must be undergoing an acceleration and therefore not have a constant velocity. Two non-inertial frames will also be accelerating with respect to each other, unless they are accelerating in exactly the same way and in the same direction.
7. Velocities don't add up like they do in Newtonian mechanics. The relativity of space and time extends to velocity. The two velocities can be added together and the relative velocity for this data will be  $(0.70+0.70)c/(1+0.70 \times 0.70) = 1.4c/1.49 = 0.94c$ .
8. The Lorentz contraction is not significant for an aircraft flying at a speed well below the velocity of light so there are no design issues to consider.  
 It is not necessary to worry about Lorentz contraction in any ship design, regardless of speed, because such contraction is relative and measured from a frame external to that of the ship. In the ship's own frame it has its proper length.
9. Density is mass per unit volume. Relativistic speed will result in a relativistic mass increase and a relativistic length decrease, therefore a relativistic increase in density.
10. When viewed in the reference frame of the starship the distance between the Earth and Alpha Centauri is seen as being length contracted and the clock as running normally.  
 To the Earth-based observer the spaceship clock is seen as being time dilated by the same factor but the distance is unchanged.  
 Observers in both locations agree the journey time is the same relative to their own frames of reference but their reasons are quite different –one being due to time dilation and the second to length contraction.
11. The relativistic mass of an object clearly increases as velocity increases. The length also decrease as at relativistic speeds. If length contracts then volume must decrease and as density = mass/volume, an increase in mass and a decrease in volume must result in an increase in density.

12. a) Length and time contract making  $c = Dx/Dt$  constant
- b) An observer outside would see that the spaceship would have changed its shape, being shorter in the direction of travel. However, in the spaceship's frame, everything has its proper length.
13. To a photon of light distance and time do not exist – so light is in effect subject to the effects mentioned
14. Light within a prism or some other medium appears to travel slower – this is due to the interaction of the electromagnetic wave with electric and magnetic fields within the material.
15. 
$$L = L_0 \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2.2 \times \frac{1}{\sqrt{1 - 0.49}} = 3.1 \text{ km}$$
16. According to you (in the spaceship), your clock runs exactly the same as it did when you were at rest on Earth, all objects in your ship appear the same to you as they did before, and the speed of light is still  $c$ . There is nothing you can do to find out if you are actually moving.
17. The speed of light is the same in all reference frames, independent of the speed of the source or the observer. Therefore, the light still travels at the speed  $c$ , and what you see in the mirror will be exactly the same as what you would see if you were at rest.
18. The scientist will observe the light beam reaching him at speed =  $c$ . Because of the principle of the constancy of the velocity of light, each observer will measure the light beam from the headlight as traveling at the same speed. This may be contrary to what you expected as you might have thought that the observer in oncoming spaceship would have measured the beam moving at (the speed of light) + (2 x the speed of the spaceship). Nevertheless, this is not what is observed in practice. What actually occurs in the real world is that no one ever measures light moving faster or slower than  $c$ .